

Further Pure 1 - January 2007

① a) ii) $x^2 + 16 = 0 \rightarrow x^2 = -16$
 $x = \pm\sqrt{-16} \rightarrow x = \pm 4i$

iii) $x^2 - 2x + 17 = 0$
 $\rightarrow (x-1)^2 - 1 + 17 = 0 \rightarrow (x-1)^2 = -16$
 $\rightarrow x-1 = \pm 4i$
 $\rightarrow x = 1 \pm 4i$

b) i) $(1+x)^3 = 1 + 3x + 3x^2 + x^3$

ii) $= 1 + 3i + 3i^2 + i^3$
 $= 1 + 3i - 3 - i \rightarrow -2 + 2i$

iii) $(1+i)^3 + 2(1+i) - 4i$
 $= -2 + 2i + 2 + 2i - 4i$
 $= 4i - 4i = 0$

② a) i) $A+B = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} + \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/2 \end{bmatrix}$
 $= \begin{bmatrix} \sqrt{3} & 0 \\ 1 & 0 \end{bmatrix}$

ii) $BA = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$

b) i) $\theta = 30^\circ \rightarrow$ rotation 30° anti-cw about $(0,0)$

ii) $2\theta = 30^\circ \rightarrow$ reflection in line $y = (\tan 15^\circ)x$

iii) Reflection in x -axis

③ a) $\alpha + \beta = -4/2 = -2$ $\alpha\beta = 3/2$

b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
 $= (-2)^2 - 2(3/2) = 4 - 3 = 1$

c) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$
 $= (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$
 $= 1 - 2(3/2)^2 = 1 - 9/2 = -7/2$

④ a) $y = ax^b$

$\log_{10} y = \log_{10} (ax^b)$

$\rightarrow \log_{10} y = \log_{10} (a) + b \log_{10} (x)$

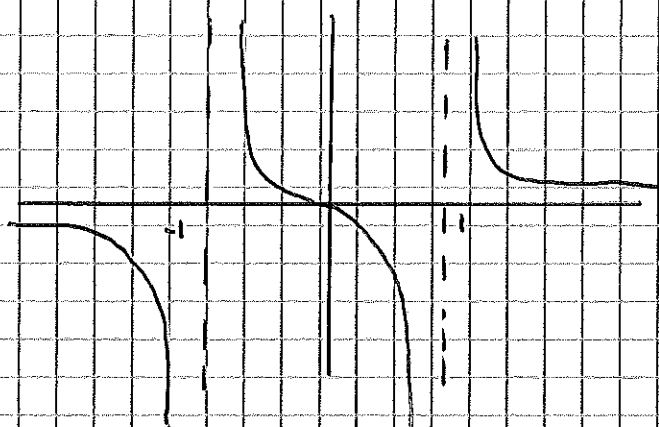
b) Gradient = $-1/2 = b$

y intercept = 1 $\rightarrow \log_{10} (a) = 1$
 $\rightarrow a = 10^1 = 10$

⑤ a) Denominator = 0 when $x = 1$ or $-1 \rightarrow x \neq 1, x \neq -1$

As $x \rightarrow \infty, y \rightarrow 0/1 \rightarrow y = 0$

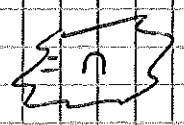
b) when $x = 0, y = 0$



c) From graphⁿ, curve lies above the x-axis when: $-1 < x < 0$ and $x > 1$

⑥ a) i) $(2r-1)^2 = 4r^2 - 4r + 1$

ii) $\sum (2r-1)^2 = 4 \sum r^2 - 4 \sum r + \sum 1$
 ~~$= 4 \cdot \frac{2}{3} n(n+1)(2n+1) - 4 \cdot \frac{1}{2} n(n+1) + n$~~
 ~~$= \frac{8}{3} n(n+1)(2n+1) - 2n(n+1) + n$~~
 ~~$= \frac{1}{3} n(n+1) [8(2n+1) - 6] + n$~~



$$\begin{aligned}
 &= \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n \\
 &= \frac{1}{3}n \left[2(n+1)(2n+1) - 6(n+1) + 3 \right] \\
 &= \frac{1}{3}n \left[2(2n^2 + 3n + 1) - 6n - 6 + 3 \right] \\
 &= \frac{1}{3}n \left[4n^2 + 6n + 2 - 6n - 6 + 3 \right] \\
 &= \frac{1}{3}n \left[4n^2 - 1 \right]
 \end{aligned}$$

b) Odd numbers are written in form $2n-1$

if $101 = 2n-1 \rightarrow n = 50$

if $199 = 2n-1 \rightarrow n = 100$

\therefore we need $\sum_{r=1}^{100} (2r-1)^2 - \sum_{r=1}^{50} (2r-1)^2$

$$\begin{aligned}
 &= \frac{1}{3}(100)(4 \times 100^2 - 1) - \frac{1}{3}(50)(4 \times 50^2 - 1) \\
 &= 1333300 - 166650 \\
 &= 1,166,650
 \end{aligned}$$

① a) G.S. $\theta = 2n\pi + a$, $\theta = 2n\pi + (\pi + a)$

key angle $(a) = \sin^{-1}(0) = 0$

$\rightarrow x + \pi/6 = 2n\pi$, $x + \pi/6 = 2n\pi + \pi$

$\rightarrow x = 2n\pi - \pi/6$, $x = 2n\pi + 5\pi/6$

b) i) $g(0.05) = 1/2 + \sqrt{3}/2(0.05) - 1/4(0.05)^2$
 $= 0.54266\dots = 0.5427$ (4dp)

$f(0.05) = \sin(0.05 + \pi/6)$
 $= 0.54266\dots = 0.5427$ (4dp)

ii) $g(h) = 1/2 + \sqrt{3}/2(h) - 1/4(h)^2$, $g(0) = 1/2$

gradient = $\frac{g(h) - g(0)}{h} = \frac{1/2 + \sqrt{3}/2(h) - 1/4(h)^2 - 1/2}{h}$
 $= \sqrt{3}/2 - 1/4h$

iii) As $h \rightarrow 0$, gradient $\rightarrow \sqrt{3}/2$

(8) a) $x = 10 \rightarrow \frac{100}{25} - \frac{y^2}{9} = 1$

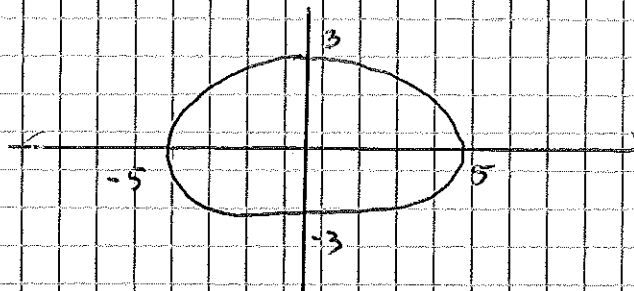
$= 4 - \frac{y^2}{9} = 1$

$\rightarrow 36 - y^2 = 9$

$\rightarrow y^2 = 27 \rightarrow y = \pm \sqrt{27} = \pm 3\sqrt{3}$

b) when $x = 0$, $\frac{y^2}{9} = 1 \rightarrow y = \pm 3$

when $y = 0$, $\frac{x^2}{25} = 1 \rightarrow x = \pm 5$



c) tangent $x = 5$

d) i) $y = x - 4 \rightarrow \frac{x^2}{25} - \frac{(x-4)^2}{9} = 1$

$x^2 - \frac{25(x-4)^2}{9} = 25$

$\rightarrow 9x^2 - 25(x-4)^2 = 225$

$\rightarrow 9x^2 - 25[x^2 - 8x + 16] = 225$

$\rightarrow 9x^2 - 25x^2 + 200x - 400 = 225$

$\rightarrow -16x^2 + 200x - 625 = 0$

$\rightarrow 16x^2 - 200x + 625 = 0$

ii) $(4x^2 - 25)(4x - 25) = 0$

$\therefore x = 25/4$

Equal roots, \therefore the line is a tangent to the curve